ERRORS OF MEASUREMENT AND 'ECOLOGICAL' CORRELATION

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1. INTRODUCTION

The term "ecological correlation" refers to the correlation coefficient based on ecological data, which means the variables describing properties of groups (e.g., averages or proportions for city blocks, enumeration districts, census tracts, etc.) [12,21].

Robinson [21] demonstrated ecological correlations cannot be used to represent individual correlations. Since then, L.A. Goodman [12] generalized Robinson's work and discussed some special cases in which ecological correlations may be used to represent individual correlations.

Despite the problems associated with ecological data, they are still used by many sociologists and scientists. The reasons for this practice may vary from case to case. Ecological data may be used because the main interest may be in studying group characteristics or the relationship between group and individual variables, or simply because the only data available are ecological data.

In most cases, the variables we observe or measure are subject to errors of measurement. Recently there has been a considerable amount of work in the study of errors of measurement [e.g., 2,3, 7,13,14,16,20,22,23].

Quite recently, Chai [4], Cochran [8], Horvitz and Koch [15], Koch [17], and Mandansky [18] have contributed to the development of the theory and application of errors of measurement in surveys beyond the univariate case.

The purpose of this paper is to study the combined effect of errors of measurement and ecological (grouped) data on estimation of the ordinary pearsonian product-moment correlation coefficient when the estimator used is based on a sample of ecological data. We present the mathematical model for the component bias factors of the estimator first and a discussion of the estimates of the component bias factors next.

2. MODEL

For the sake of simplicity, we consider a simple random sample of very large size n taken from a finite population of size N. This sample is then "interpenetrated" into M subgroups, each subgroup containing $\bar{n} = \frac{n}{N}$ elements. We assume that each of M interviewers is assigned to a subgroup and that the collection and processing of data are designed in such a way that there is no correlation between the response errors of any two units in different subgroups. This is to say that correlated errors are expected only within subgroups. We further assume that the survey is repeatable under a constant survey condition and that the finite multipliers for every subgroup

$(1-\frac{\bar{n}}{\bar{N}})$ and for the entire sample $(1-\frac{n}{N})$ can be ignored.

The model shown below, under the above simple assumptions, may be an over-simplification of the real world; but the modification and/or extension of the model for more realistic survey conditions can easily be made [e.g., 4,6,13].

Let x_{ijt} and y_{ijt}, respectively, be the sample responses for the j-th individual unit of the i-th subgroup recorded at the t-th measurement. And let $\bar{x}_{it}$ and $\bar{y}_{it}$, respectively, be the sample average responses for the i-th subgroup (group) recorded at the t-th measurement.

Following Hansen et.al., [13], we write:

$$x_{ijt} = X_{ij} - d_{ijt} \tag{1}$$

$$y_{ijt} = Y_{ij} - e_{ijt} \tag{2}$$

where X_{ij} and Y_{ij} are the conditional expected values, i.e.,

$$X_{ij} = \underset{t}{E}(x_{ijt} \mid i,j) \tag{3}$$

$$Y_{ij} = \underset{t}{E}(y_{ijt} \mid i,j) \tag{4}$$

and d_{ijt} and e_{ijt} are the "response deviations" of x_{ijt} and y_{ijt}.

Suppose that we are interested in estimating the correlation coefficient of expected values for individual units (X_{ij} and Y_{ij}), ρ ($\rho=\sigma_{XY}/\sigma_X\sigma_Y$) from a sample of grouped data.

Let the estimator of ρ be the Pearsonian product-moment formula based on the sample grouped data, i.e.,

$$r_{At} = \frac{s_{\bar{x}\bar{y}(t)}}{s_{\bar{x}(t)}s_{\bar{y}(t)}} \tag{5}$$

where $s_{\bar{x}\bar{y}(t)}$ is the between-area sample covariance observed for the t-th trial and $s_{\bar{x}(t)}$ and $s_{\bar{y}(t)}$ respectively are the between-area sample standard deviations observed for the t-th trial.

Now, let

$$\rho_A^* = \frac{\underset{st}{EE}\ s_{\bar{x}\bar{y}(t)}}{(\underset{st}{EE}\ s^2_{\bar{x}(t)}\ \underset{st}{EE}\ s^2_{\bar{y}(t)})^{1/2}} \qquad (6)$$

Then, it is shown [5][1] that, under the survey conditions assumed in this paper,

$$\rho^*_A = (\rho)\cdot(\varepsilon_1)\cdot(\varepsilon_2) \qquad (7)$$

where ε_1, the component bias factor due to errors of measurement only, is defined by

$$\varepsilon_1 \doteq \frac{1+\sigma_{de}/\sigma_{XY}}{[(1+\sigma^2_d/\sigma^2_X)(1+\sigma^2_e/\sigma^2_Y)]^{1/2}}; \qquad (8)$$

and ε_2, the component factor due to grouping and interaction between errors of measurement and grouping, is defined by

$$\varepsilon_2 \doteq (\rho_A/\rho)\cdot(1/\varepsilon_1)\cdot \frac{(1+\sigma_{de(B)}/\sigma_{XY(B)})}{[(1+\sigma^2_{d(B)}/\sigma^2_{X(B)})(1+\sigma^2_{e(B)}/\sigma^2_{Y(B)})]^{1/2}} \qquad (9)$$

where σ^2_d, σ^2_e and σ_{de} respectively are "simple response variance and covariance" [13]; σ^2_X, σ^2_Y, and σ_{XY} are the variance and covariance of expected values for ungrouped data; $\sigma^2_{d(B)}$, $\sigma^2_{e(B)}$, and $\sigma_{de(B)}$ are the variance and covariance of the response deviations for grouped data [2]; $\sigma^2_{X(B)}$, $\sigma^2_{Y(B)}$, and $\sigma_{XY(B)}$ are the variance and covariance of expected values for grouped data: and $\rho_A=\sigma_{XY(B)}/\sigma_{X(B)}\sigma_{Y(B)}$ is the ecological correlation coefficient for expected values (see reference [5] for further details).

Let, the third term of Equation (9) above be denoted by ε_3. We call ε_3 the bias component factor due to grouping. Furthermore, we define the interaction term, I by the ratio of ε_3 to ε_1, i.e.,

$$I = \varepsilon_3/\varepsilon_1 \qquad (10)$$

Then, from Equation (9) we have

$$\varepsilon_2 = (\rho_A/\rho)\cdot(I) \qquad (11)$$

and from Equation (7) we have

$$\rho^*_A = (\rho_A)\cdot(\varepsilon_3) \qquad (12)$$

If I = 1 (no effect due to interaction), then we have $\rho^*_A = (\rho_A)\cdot(\varepsilon_1)$ (13)

We may express Equation (7) above using the definition

$$\rho^* = \underset{st}{EE}\ s_{xy(t)}/[(\underset{st}{EE}\ s^2_{x(t)})\cdot(\underset{st}{EE}\ s^2_{y(t)})]^{1/2} = (\rho)\cdot(\varepsilon_1) \qquad (14)$$

where $s^2_{x(t)}$, $s^2_{y(t)}$, and $s_{xy(t)}$ respectively are the sample variance and covariance of observed values for ungrouped data. Hence,

$$\rho^*_A = (\rho^*)\cdot(\varepsilon_2) \qquad (15)$$

In summary, we have for grouping effect only:

$$\rho_A/\rho = (\varepsilon_1)\cdot(\varepsilon_2)/(\varepsilon_3) \qquad (16)$$

combined effect of errors of measurement and grouping:

$$\rho^*_A/\rho = (\rho_A/\rho)\cdot(\varepsilon_3) \text{ if } I \neq 1 \qquad (17.1)$$

$$\rho^*_A/\rho = (\rho_A/\rho)\cdot(\varepsilon_1) \text{ if } I = 1 \qquad (17.2)$$

3. ESTIMATION OF COMPONENT BIAS FACTORS

A detailed discussion of the estimation procedures used to estimate ε_1, ε_2 and ε_3 are given in reference [5] and a brief summary of the estimators used is given in the Appendix of this paper.

The sample estimates of the bias factors are calculated for some housing variables and are summarized in Table 1. These estimates are obtained from two different sources--(1) the 1960 Census of Population and Housing as the original data and a probability sample of 5000 housing units located in approximately 2500 area segments of the United States in October 1960 (six months after the 1960 Census) for reinterview purposes [22] and (2) the six-city sample data used for the purposes of evaluating the quality of housing units at the Bureau of the Census in 1964-65 [23].

The first set of sample data is used primarily to estimate the simple response variance components $(\sigma^2_d/\sigma^2_X, \sigma^2_e/\sigma^2_Y)$ and the covariance component $(\sigma_{de}/\sigma_{XY})$, and the second sample data was used exclusively to estimate the averages of the correlated component of response variances $(\bar{\Delta}_d, \bar{\Delta}_e)$

and covariance $(\bar{A}_{de})$[3/]. The ecological data used are for city blocks, enumeration districts (ED), and census tracts.

4. DISCUSSION

First we discuss the grouping effect only (ρ_A/ρ) (see column 3 of the table) and secondly we study the combined effect of errors of measurement and grouping (ρ_A^*/ρ) (see columns 6 and 7 of the table).

Grouping Effect

The estimates of ρ_A^*/ρ^* estimated by r_{At}/r_t (see Appendix) in Column 2 of the table reflect the estimates of the ratio of the ecological correlation to individual correlation based on observed values, which, of course, are subject to errors of measurement; whereas the estimates of ρ_A/ρ show the grouping effect only (no errors of measurement are included).

The estimates of ρ_A^*/ρ^* show results that are quite similar to the ones given by the earlier experimental works (Gehlke and Biehl [10], Robinson [21], Duncan and Davis [9], Abel and Waugh [1] and Pritzker and Selove [23]). In other words, the earlier works showed that (1) the estimates of ρ_A^* are greater than ρ^* (i.e., $\rho_A^*/\rho^*>1$) and that (2) the estimates of ρ_A^* are greater for a large group than for a small group.

However, the estimates of ρ_A/ρ do not necessarily follow the same patterns as the ones given by the estimates of ρ_A^*/ρ^*. We note first that the estimates of ρ_A/ρ are smaller than the estimates of ρ_A^*/ρ^* for most of the cases given in this study.

This is evident, since

$$\frac{\rho_A^*}{\rho^*} = \left(\frac{\rho_A}{\rho}\right)\cdot\left(\frac{\varepsilon_3}{\varepsilon_1}\right) = \left(\frac{\rho_A}{\rho}\right)\cdot(I)$$

and the estimates of the interaction term I are significantly greater than one for most cases (Column 5). In fact, a comparison of the estimates of ρ_A/ρ with those of "I" reveals the interaction effect to be stronger than the grouping effect.

Furthermore, we note that, unlike the estimates of ρ_A^*/ρ^*, half of the estimates of ρ_A/ρ given in this paper does not increase as the area size increases. This seems to imply that the ecological correlation in the absence of errors of measurement has an attenuating effect on the ordinary estimator of $\rho(r_{At})$ rather than an inflating effect as was indicated by the earlier works.

Also, the estimates of "I" given in this paper definitely cast some doubts on the possibility that "random errors" (Yule and Kendall [24] may cancel out when individual units are grouped and when the size of the group increases.

Combined Effects [See Equations (7), (17.1), and (17.2)

Columns 6 and 7 show the estimates for the combined effect $(\varepsilon_1.\varepsilon_2)$. The estimates of $(\varepsilon_1.\varepsilon_2)$ are greater than one for all cases except one, meaning that the estimator r_{At} over-estimates ρ, on the average, for most of the cases studied.

Furthermore, the estimates of $(\varepsilon_1.\varepsilon_2)$ for larger-size groups are greater than those for **small**-sized groups. This indicates that the bias due to errors of measurement and grouping are increasing as the size of ecological groups increases.

It is interesting to compare the estimates of $\varepsilon_1=\rho^*/\rho$, component bias factor due to errors of measurement when no grouping is made (Column 1), with the estimates of $\varepsilon_2=(\rho_A^*/\rho).(I)$ (Column 2); for the estimates of ε_2 are greater than the ones for ε_1 in practically all of the cases considered. This, of course, suggests that the grouping (ρ_A/ρ) and interaction (I) effects are greater than the effect due to errors of measurement alone.

The estimates of the component bias factors presented above simply illustrate that the estimation of the individual correlation ρ using the Pearsonian product-moment estimator based on ecological data (r_{At}) is affected not only by grouping error but also by errors of measurement and by the interaction of the two. Although more study based on more variables are needed, this study clearly demonstrates the possible bias due to errors of measurement and to the use of the estimator of the ecological correlation coefficient for estimation of the individual (ungrouped) correlation coefficient.

Appendix
ESTIMATORS USED

Detailed account of estimation procedures is given by reference [5]. Only a brief summary of the estimators used is given below.

1. Estimator of ε_1,

To estimate ε_1, the factors σ_{de}/σ_{XY}, σ_d^2/σ_X^2 (or σ_e^2/σ_Y^2) must be estimated. The estimator used for σ_d^2/σ_X^2 (or σ_e^2/σ_Y^2) is [4]:

$$\left(\frac{g}{2s^2_{x(T)}}\right)\cdot\left(1-\frac{g}{2s^2_{x(T)}}\right)^{-1} \quad \text{(A-1)}$$

where

$$g = \frac{1}{n}\sum_i^M\sum_j^{\bar{n}}(x_{ijt}-x_{ijt'})^2 \quad \text{(A-2)}$$

is the "gross difference rate" [14] and is the estimator of σ_d^2: and

$$s^2_{x(T)} = \frac{1}{2}(s^2_{xt}+s^2_{xt'}) \quad \text{(A-3)}$$

is the estimator of $\sigma^2_{x(T)} \doteq \sigma_X^2 + \sigma_d^2$

Hence, $\frac{g}{2s^2_{x(T)}}$ is the estimator of $\sigma_d^2/\sigma^2_{x(T)}$, "index of inconsistency" [14]. The estimator of σ_{de}/σ_{XY} is [4]:

$$\left(\frac{h}{2s_{xy(T)}}\right)\cdot\left(1-\frac{h}{2s_{xy(T)}}\right)^{-1} \quad \text{(A-4)}$$

where

$$h = \frac{1}{n}\sum_i^M\sum_j^{\bar{n}}(x_{ijt}-x_{ijt'})(y_{ijt}-y_{ijt'}) \quad \text{(A-5)}$$

is the estimator of σ_{de} (see reference [20]) and $s_{xy(T)}$ [see (A-3) above] is the estimator of $\sigma_{xy(T)} \doteq \sigma_{XY} + \sigma_{de}$.

2. Estimator of ε_2

Noting that

$$\varepsilon_2 = \rho^*_A/\rho^*$$

we use

$$\frac{r_{At}}{r_t} \quad \text{(A-6)}$$

as the estimator of ε_2. Where

$$r_t = \frac{s_{xy(t)}}{s_{x(t)}s_{y(t)}} \quad \text{(A-7)}$$

3. Estimator of ε_3

To estimate ε_3, the factors $\sigma_{de(B)}/\sigma_{XY(B)}$ and $\sigma^2_{d(B)}/\sigma^2_{X(B)}$ (or $\sigma^2_{e(B)}/\sigma^2_{Y(B)}$) must be estimated,

where $\sigma_{de(B)}$, $\sigma^2_{d(B)}$, and $\sigma^2_{e(B)}$ are given by $\bar{\Delta}_{de}$, $\bar{\Delta}_d$, $\bar{\Delta}_e$ (see footnote 2).

The estimator of $\bar{\Delta}_d/\sigma^2_{X(B)}$ (or $\bar{\Delta}_e/\sigma^2_{Y(B)}$) is:

$$\left(\frac{\hat{\bar{\Delta}}_d}{s^2_{\bar{x}(t)}}\right)\cdot\left(1-\frac{\hat{\bar{\Delta}}_d}{s^2_{\bar{x}(t)}}\right)^{-1} \quad \text{(A-8)}$$

where

$$\hat{\bar{\Delta}}_d = \frac{\bar{n}\,(s^2_{\bar{x}(t)} - s^2_{x(t)}/\bar{n})}{\bar{n}-1} \quad \text{(A-9)}$$

is the estimator of $\hat{\bar{\Delta}}_d$ and $s^2_{\bar{x}(t)}$ is the estimator of $\sigma^2_{x(B)(T)} \doteq \sigma^2_{X(B)} + \bar{\Delta}_d$

The estimator of $\bar{\Delta}_{de}/\sigma_{XY(B)}$ is:

$$\left(\frac{\hat{\bar{\Delta}}_{de}}{s_{\bar{x}\bar{y}(t)}}\right)\cdot\left(1-\frac{\hat{\bar{\Delta}}_{de}}{s_{\bar{x}\bar{y}(t)}}\right)^{-1} \quad \text{(A-10)}$$

where

$$\hat{\bar{\Delta}}_{de} = \frac{\bar{n}\,(s_{\bar{x}\bar{y}(t)} - s_{xy(t)}/\bar{n})}{\bar{n}-1}$$

is the estimator of $\bar{\Delta}_{de}$ and $s_{\bar{x}\bar{y}(t)}$ is the estimator of $\sigma_{xy(B)(T)} \doteq \sigma_{XY(B)} + \bar{\Delta}_{de}$

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are those of the author and not necessarily those of the Bureau of the Census.

1/ Reference [5] shows that:

$$0[{}^{EE}_{st}(r_{At} - \rho^*_A)] \doteq 0(\frac{1}{M})$$

2/ For the survey conditions defined here,

$\sigma^2_{d(B)} \doteq \bar{\Delta}_d$, $\sigma^2_{e(B)} \doteq \bar{\Delta}_e$, and $\sigma_{de(B)} \doteq \bar{\Delta}_{de}$, where $\bar{\Delta}_d$, $\bar{\Delta}_e$, and $\bar{\Delta}_{de}$ respectively are the average of the correlated component of the response variance and covariance (the correlated component of the response variance and covariance are based on the intraclass correlation coefficient of response deviation for subgroups).

3/ Although the sample data used to estimate $\bar{\Delta}_d$, $\bar{\Delta}_e$, and $\bar{\Delta}_{de}$ are different from the sample estimating the simple response variance and covariance components, the estimates of $\bar{\Delta}_d$ and $\bar{\Delta}_e$ obtained from the six-city data seem to show the order of magnitudes and the patterns of variation for the different ecological groups similar to the ones estimated at the Census Bureau for other variables based on a much larger scale survey [2,5].

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TABLE 1. ESTIMATES OF COMPONENT BIAS FACTORS

City and Variables	Estimate of: $\varepsilon_1=\rho^*/\rho$	$\varepsilon_2=\rho^*_A/\rho^*$			$\rho_A/\rho=\varepsilon_1\cdot\varepsilon_2/\varepsilon_3$			$\varepsilon_3=\rho^*_A/\rho_A$			$I=\varepsilon_3/\varepsilon_1$			$\varepsilon_1\cdot\varepsilon_2=\rho^*_A/\rho$ when $I\neq 1$			ρ^*_A/ρ when $I=1$		
		Block	ED	Tract	Block	ED	Tract	Block	ED	Tract	Block	ED	Tract	Block	ED	Tract	Block	ED	Tract
Units deteriorating vs. Owner occupied units:																			
Camden, New Jersey	1.05	1.01	1.30	3.67	.28	.87	1.14	3.73	1.57	3.37	3.55	1.50	3.21	1.06	1.36	3.85	.29	.91	1.20
Cleveland, Ohio	1.05	2.05	2.34	3.54	.40	.51	.42	5.31	4.87	8.85	5.06	4.64	8.43	2.15	2.46	3.72	.42	.54	.44
Fort Wayne, Indiana	1.05	2.00	2.88	3.88	.58	1.73	.80	3.64	1.75	5.06	3.47	1.67	4.82	2.10	3.02	4.07	.61	1.82	.84
Louisville, Kentucky	1.05	1.46	1.88	2.84	.71	.64	.67	2.14	3.07	4.42	2.04	2.92	4.21	1.53	1.97	2.98	.75	.67	.70
Shreveport, Louisiana	1.05	1.58	2.00	2.58	.47	.43	.45	3.53	4.86	6.02	3.36	4.63	5.73	1.66	2.10	2.71	.49	.45	.47
South Bend, Indiana	1.05	1.74	2.36	3.67	.52	.66	1.60	3.49	3.74	2.41	3.32	3.56	2.30	1.83	2.48	3.85	.55	.69	1.68
AVERAGE	1.05	1.64	2.13	3.36	.49	.81	.85	3.64	3.31	5.02	3.47	3.15	4.78	1.72	2.24	3.53	.51	.85	.89
Units deteriorating vs. Units with both for exclusive use:																			
Camden, New Jersey	.66	1.72	2.33	3.55	.29	.30	**	3.90	5.04	**	5.90	7.64	**	1.14	1.54	2.34	.19	.20	**
Cleveland, Ohio	.66	1.79	2.11	2.93	.34	.23	.59	3.56	6.09	3.33	5.39	9.23	5.05	1.18	1.39	1.93	.22	.20	.39
Fort Wayne, Indiana	.66	1.19	2.08	3.00	.52	.64	1.20	1.33	2.12	1.64	2.02	3.21	2.48	.79	1.37	1.98	.34	.42	.79
Louisville, Kentucky	.66	1.95	2.50	3.03	.25	.32	.88	5.03	4.89	2.30	7.62	7.41	3.48	1.29	1.65	2.00	.16	.21	.58
Shreveport, Louisiana	.66	1.65	2.12	2.77	.18	.25	.55	6.18	5.71	3.32	9.36	8.65	5.03	1.09	1.40	1.83	.12	.16	.36
South Bend, Indiana	.66	1.78	2.71	4.04	.18	.24	**	6.50	7.60	**	9.85	11.51	**	1.17	1.79	2.67	.12	.16	**
AVERAGE	.66	1.68	2.31	3.22	.29	.33	.80	4.42	5.24	2.65	6.69	7.94	4.01	1.11	1.52	2.12	.19	.22	.53

** Undefined, since the estimates of $\bar{\Delta}_{de}/\sigma_{xy(B)} \doteq 1$

Source: References [4 and 5]